LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **APRIL 2023**

PST 3601 – ADVANCED OPERATIONS RESEARCH

Date: 08-05-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION A

Answer ALL questions. Each carries two marks. **(10 x 2 = 20)**

1. Define a Generalized Linear Programming Problem.
2. What is the need for integer programming?
3. Define Pure integer programming problem.
4. Show that dual of dual is primal for the following LPP; Maximize $Z = 2X_1 + X_2$, subject to the constraints, $X_1 + 2X_2 \leq 10$; $X_1 + X_2 \leq 6$; and $X_1, X_2 \geq 0$.
5. What is the basic principle used in Dynamic Programming?
6. Define Non Linear Programming Problem.
7. Give the mathematical representation of a QPP.
8. When do we say that a function is convex or concave?
9. What is scientific inventory management?
10. What do you mean by jockeying in a queue?

SECTION B

Answer any FIVE questions. Each carries eight marks. **(5 x 8 = 40)**

11. Use Two-phase simplex method to solve the following LPP, Maximize $Z = 4x + 5y$ subject to $2x + 3y \leq 6$, $3x + y \geq 3$, $x, y \geq 0$.
12. Describe the Gomory's constraint method, and derive Gomory's constraint for solving a Pure Integer Programming Problem.
13. Solve the NLPP, $z = 4x_1 + 8x_2 - x_1^2 - x_2^2$, $z = 4x_1 + 8x_2 - x_1^2 - x_2^2$ subjected to $x_1 + x_2 = 4$, $x_1, x_2 \geq 0$.
14. Solve the following mixed integer programming problems using Gomory's cutting plane method by using the initial solution given below; Maximize $z = x_1 + x_2$ subject to constraints $3x_1 + 2x_2 \leq 5$, $x_2 \leq 2$, $x_1, x_2 \geq 0$ and x_1 is integer.

    Introducing slack variables and using simplex method, a non-integer optimum solution is given below;

C	X_B	X_0	X_1	X_2	X_3	X_4
1	X_1	1/3	1	0	1/3	-2/3
1	X_2	2	0	1	0	1
	Z-C	7/3	0	0	1/3	1/3

Find an OBFS to the above problem.

15. Write the necessary conditions to solve the following Quadratic programming Problem, Min $Z = X_1 - 3 X_2 - 5 X_3 + 2 X_1 X_2 + 2 X_2 X_3 + 2 X_1^2 + 2 X_2^2 + 3 X_3^2$ subject to the constraints, $X_1 + X_2 + X_3 \leq 1$; $3 X_1 + 2 X_2 + X_3 \leq 6$;

and $X_1 , X_2 , X_3 \geq 0$.

16. Explain the characteristics and the algorithm of solving a Dynamic Programming Problem.

17. Describe the components of an inventory model.

18. Explain the elements of a queuing system.

SECTION C

Answer any TWO questions. Each carries twenty marks. **(2 x 20 = 40)**

19. Find an optimum integer solution for the following LPP: Max $Z = X_1 + 2 X_2$, subject to the constraints, $X_1 + X_2 \leq 7$; $2 X_1 \leq 11$; $2 X_2 \leq 7$ and X_1 , X_2 are non-negative integers.

20. Solve the following Non Linear Programming Problem: Max $Z = 2 X_1 - X_1^2 + X_2$ subject to the constraints, $2 X_1 + 3 X_2 \leq 6$; $2 X_1 + X_2 \leq 4$; and $X_1 , X_2 \geq 0$.

21. Derive the Khun-Tucker Necessary conditions for solving a Generalized Non-Linear Programming Problem for solving a Maximization/Minimization (both cases) objective function with one constraint.

22. For a (M/M/1) : (∞/FIFO) queuing model in the steady-state case, derive the steady state difference equations and obtain expressions for the mean and variance of queue length in terms of the parameters λ and μ.

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